Perfect substitutes

Consumer theory · Problem 1 · Video: youtu.be/qKRKmEKNqFQ

Solved problem

During exam week a student wants only to stay awake, and treats tea and coffee as interchangeable sources of caffeine. Take one cup of coffee to carry twice the caffeine of one cup of tea, and let

- x: cups of tea, at $p_x = 4$ each;
- y: cups of coffee, at $p_y = 5$ each.

Total caffeine, measured in cups-of-tea, is then $u(x, y) = x + 2y$. With a weekly budget of \$20 for hot drinks, find the optimal bundle $(x^\star, y^\star)$ and illustrate the budget line and the optimal choice in a diagram.

Solution

The student solves

$$\begin{aligned} \max_{(x,y)\in\mathbb{R}^2_+} \quad & x + 2y \\ \text{s.t.} \quad & 4x + 5y \le 20. \end{aligned}$$

Compare the marginal utility per dollar spent on each good:

$$\frac{\mathrm{MU}_x}{p_x} = \frac{1}{4} \qquad \text{and} \qquad \frac{\mathrm{MU}_y}{p_y} = \frac{2}{5}.$$

Since $\mathrm{MU}_x/p_x < \mathrm{MU}_y/p_y$, every dollar buys more caffeine from coffee, so the student spends the whole budget on y (coffee). With $M = 20$ and $p_y = 5$, the optimal bundle is

$$(x^\star, y^\star) = \left(0, \tfrac{M}{p_y}\right) = (0, 4).$$

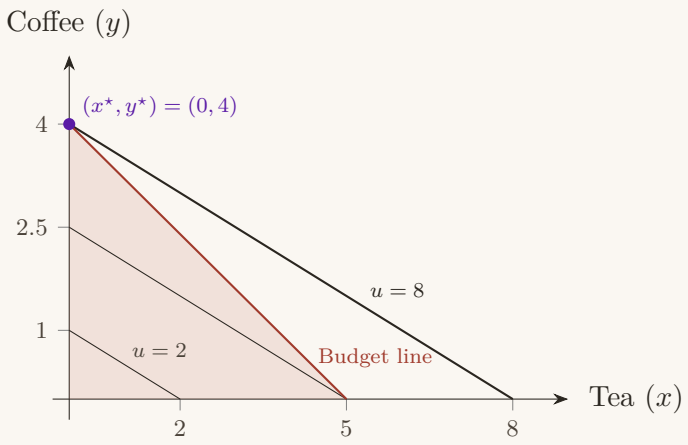

Figure 1. Indifference curves are straight lines $x + 2y = \bar{u}$ with absolute slope $\mathrm{MU}_x/\mathrm{MU}_y = 1/2$. The budget line has absolute slope $p_x/p_y = 4/5$ and is steeper, so the highest attainable indifference curve is reached at the corner $(0, 4)$, where the budget line meets the y-axis.

Exercise

A student commutes to university by either:

- x: bus rides;
- y: metro rides.

Each ride, by either mode, is one trip to university, so the student treats the two modes as perfect substitutes:

$$u(x, y) = x + y.$$

A bus ride costs $p_x = 2$ and a metro ride $p_y = 3$, and the weekly transport budget is \$18.

1. Draw the budget set and the budget line.
2. Compute the marginal utility per dollar spent on each mode, $\dfrac{\mathrm{MU}_x}{p_x}$ and $\dfrac{\mathrm{MU}_y}{p_y}$. Find the utility-maximising bundle $(x^\star, y^\star)$.
3. Suppose the metro authority introduces a discounted student fare, cutting the metro price to $p_y = 2$. With everything else unchanged, characterise the set of utility-maximising bundles.

Extension: quality-adjusted travel

Suppose instead the student finds the metro more convenient, because it is faster and more reliable, so preferences become

$$u(x, y) = x + 1.5\, y,$$

with the original prices and budget, $p_x = 2$, $p_y = 3$, and $M = 18$.

4. Solve for the utility-maximising bundle(s) under these preferences.

Checking your work. To discuss the exercise, join the community forum at econschool.in.