Perfect complements

Consumer theory · Problem 2 · Video: youtu.be/S4v03C39jAI

Solved problem

A student is making pancakes for the week. Each pancake uses one cup of flour, and a cup of milk is enough for two pancakes; flour with no milk, or milk with no flour, makes no more pancakes. Writing

- x: cups of flour, at $p_x = 4$ each;
- y: cups of milk, at $p_y = 4$ each,

the number of pancakes is $u(x, y) = \min\{x, 2y\}$. With a budget of \$18 for ingredients, find the optimal bundle $(x^\star, y^\star)$ and illustrate the budget line and the optimal choice in a diagram.

Solution

The student solves

$$\max_{(x,y)\in\mathbb{R}^2_+} \quad \min\{x, 2y\}$$
$$\text{s.t.} \quad 4x + 4y \le 18.$$

Extra flour does nothing without the milk to go with it, and extra milk does nothing without the flour. So at the optimum nothing is wasted,

$$x = 2y,$$

and the budget binds. Substituting $x = 2y$ into $4x + 4y = 18$,

$$4(2y) + 4y = 12y = 18 \quad \Longrightarrow \quad y = 1.5, \qquad x = 2y = 3.$$

The optimal bundle is $(x^\star, y^\star) = (3, 1.5)$, giving $u = \min\{3, 3\} = 3$ — three pancakes.

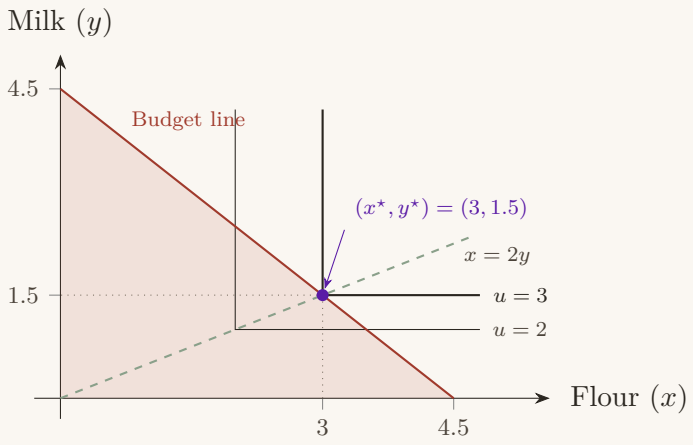

Figure 1. With perfect complements the indifference curves are L-shaped, their corners lying on the ray $x = 2y$ where flour and milk are in recipe proportion. The best the budget reaches is the corner of the highest L — on the budget line and on the ray at once — so nothing is wasted: $(x^\star, y^\star) = (3, 1.5)$.

Exercise

A student eats dal and rice in the canteen. One serving of dal with one serving of rice makes a plate; a serving of dal with no rice, or rice with no dal, is no use. Writing

- x: servings of dal, at $p_x = 2$ each;
- y: servings of rice, at $p_y = 1$ each,

the number of plates is $u(x, y) = \min\{x, y\}$. The student has \$15 to spend.

1. On a diagram, draw the budget set, the budget line, the ray of bundles that waste nothing, and a couple of indifference curves.
2. Find the utility-maximising bundle $(x^\star, y^\star)$ and the number of plates.

Extension: a price rise

Suppose a poor harvest pushes the price of dal up to $p_x = 4$, with the rice price and the \$15 budget unchanged.

3. Find the new optimal bundle and the number of plates.

Checking your work. To discuss the exercise, join the community forum at econschool.in.