Hybrid preferences

Consumer theory · *Problem 4* · *Video: youtu.be/DYMhGCBVAOE*

Solved problem

A student buys bread and butter. A slice of bread is worth something on its own; a pat of butter is worth something only spread on a slice — one pat per slice — and a buttered slice is worth twice a plain one. Butter with no bread does nothing. Writing

- x: pats of butter, at price p_x;
- y: slices of bread, at price p_y,

tastes are $u(x, y) = \min\{x + y, 2y\}$, a hybrid of substitutes and complements. For every (p_x, p_y, M), determine the demand correspondence $(X^d, Y^d)(p_x, p_y, M)$.

Solution

The student solves

$$\max_{(x,y)\in\mathbb{R}^2_+} \quad \min\{x+y,\, 2y\} \qquad \text{s.t.} \quad p_x x + p_y y \le M.$$

First read the utility. Since $x + y \le 2y$ exactly when $x \le y$,

$$u(x,y) = \min\{x+y,\, 2y\} = y + \min\{x,y\} = \begin{cases} x+y & x \le y, \\ 2y & x \ge y. \end{cases}$$

So a pat of butter only helps while some slice is still plain ($x < y$); once every slice is buttered ($x \ge y$) extra butter is wasted. The student would never pay for wasted butter, so the optimum has $x \le y$, where $u = x + y$ and butter and bread are perfect substitutes at rate $1 : 1$. It remains to maximise $x + y$ on the budget subject to $x \le y$, which turns on the cost per unit of utility, p_x for butter against p_y for bread.

If $p_x > p_y$, bread is the cheaper source of utility — a buttered slice costs $p_x + p_y$ for 2 units, beaten by two plain slices at $2p_y$ — so the student buys only bread, $(0, M/p_y)$. If $p_x < p_y$, butter is cheaper and the student wants all of it, but butter past $x = y$ is wasted, so the choice is pinned to the kink $x = y$, giving $(\frac{M}{p_x+p_y}, \frac{M}{p_x+p_y})$. If $p_x = p_y$, then $x + y = M/p_y$ at every bundle on the budget with $x \le y$, so all of them are optimal at once. Collecting these,

$$(X^d, Y^d)(p_x,p_y,M) = \begin{cases} \{(0,\, M/p_y)\} & p_x > p_y, \\ \{(x,y)\in\mathbb{R}^2_+ : p_x x + p_y y = M,\ y \ge x\} & p_x = p_y, \\ \left\{\left(\frac{M}{p_x+p_y},\, \frac{M}{p_x+p_y}\right)\right\} & p_x < p_y. \end{cases}$$

Notice that when $p_x = p_y$, the utility-maximization problem has multiple solutions, so demand here is a correspondence, or set-valued function.

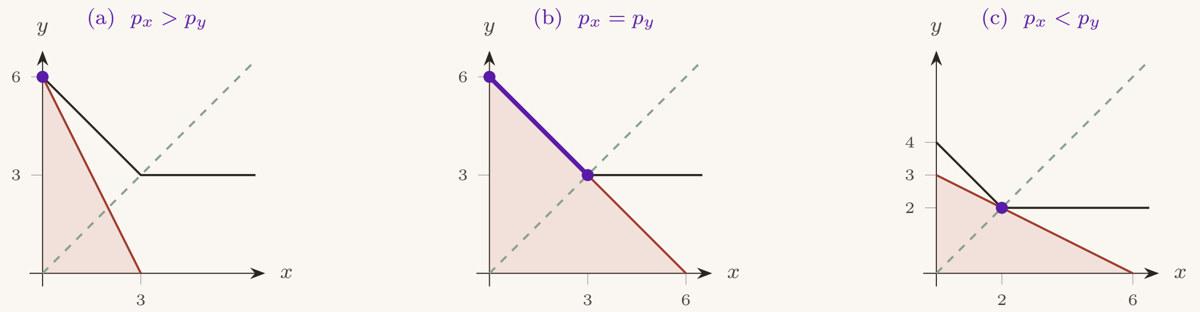

*Figure 1. The three price cases, all at $M = 6$ on common axes. The dashed line is $x = y$, where the indifference curves kink; the violet mark is the optimum. **(a)** $p_x > p_y$ ($p_x = 2$, $p_y = 1$): the budget is steeper than the slope-1 arm, so it reaches the highest indifference curve only at the bread corner $(0, 6)$. **(b)** $p_x = p_y$ ($= 1$): the slope-1 arm lies along the budget, so every bundle from $(0, 6)$ to the kink $(3, 3)$ is optimal — the violet segment. **(c)** $p_x < p_y$ ($p_x = 1$, $p_y = 2$): the budget is flatter than the sloped arm but steeper than the flat arm, so it supports the kink $(2, 2)$.*

Exercise

A student buys rice and beans. Each is filling on its own, and a serving of each eaten together makes a balanced plate that is worth an extra unit. Writing

- x: servings of rice, at price p_x;
- y: servings of beans, at price p_y,

tastes are

$$u(x, y) = \min\{2x + y,\ x + 2y\} = (x + y) + \min\{x, y\}.$$

1. Find the demand correspondence $(X^d, Y^d)(p_x, p_y, M)$. *Hint:* $u = 2x + y$ when $x \leq y$ and $u = x + 2y$ when $x \geq y$, with the kink on $x = y$.
2. Take $p_x = 3$, $p_y = 2$, $M = 12$, and find the optimal bundle. Then take $p_x = 4$, $p_y = 2$ with $M = 12$ and describe the optimal set.

Checking your work. To discuss the exercise, join the community forum at econschool.in.